The Geometric Construction Abilities Of Gifted Students In Solving Real - World Problems: A Case From Turkey

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ABSTRACT

Geometric constructions have already been of interest to mathematicians. However, studies on geometric construction are not adequate in the relevant literature. Moreover, these studies generally focus on how secondary school gifted students solve non-routine mathematical problems. The present study aims to examine the geometric construction abilities of ninth-grade (15 years old) gifted students in solving real-world geometry problems; thus a case study was conducted. Six gifted students participated in the study. The data consisted of voice records, solutions, and models made by the students on the GeoGebra screen. Results indicate that gifted students use their previous knowledge effectively during the process of geometric construction. They modeled the situations available in the problems through using mathematical concepts and the software in coordination. Therefore, it is evident that gifted students think more creatively while solving problems using GeoGebra.

Keywords: *Gifted students, Geometric construction ability, Real-world problems, Problem solving, GeoGebra software*

INTRODUCTION

NCTM (2000) emphasizes that geometric construction refers to meaningful learning. Geometric construction helps students realize the relations among different geometric models and develops their problem solving abilities (Napitupulu, 2001; Posamentier, 2000). Mathematics teaching and problem solving should be considered together in contemporary education. It is significant to determine students' problem solving abilities as well as developing them. It becomes much more significant that gifted students are considered as different from ordinary students.

Educators define the concept of giftedness in various ways as gifted students have different abilities. According to the studies carried out in mathematics education, gifted students are successful in problem solving processes such as organizing materials, using templates and rules, modifying the problem statement, using new expressions in templates and rules, understanding and studying on complex issues, reversing the processes and finding relevant problems (Gross, 2004; Lupkowski-Shoplik, Benbow, Assouline & Brody, 2003; Miller, 1990; VanTassel-Baska, 2000). Gifted students are characterized by their abilities to pay continuous attention to problem solving and their tendency to question, to test, and to explore (Johnsen, 2004). Their problem solving skills are much better than those of non-gifted students (Knepper, Obrzut, & Copeland, 1983). Gifted students tend to be more successful in solving mathematical problems than their non-gifted peers (Gallagher, 1975; Renzulli, 1978). Furthermore, gifted students use mathematical skills as effectively as older students do (Sowell, Zeigler, Bergwell, & Cartwright, 1990).

It is important to state gifted students' learning process efficiently. The study by Yıldız, Baltacı, Kurak, and Güven (2012) found that gifted students make decisions in a more different and faster way than the other students. They proceed fast, not step-by-step, skipping conceptually; furthermore, they find complicated connections among the parts by seeing the subject as a whole (Holton & Gaffney, 1994; Heward & Orlansky, 1980). Thus, gifted students should be given different sources of activities rather than given the unnecessary repetitions like other students (Meyen & Skrtic, 1988). In this manner, Kirk and Gallegher (1989) expressed that the learning process of gifted students should be faster than the other students', and they should be presented various kinds of materials.

Students need to utilize their intuitive knowledge and experiences in order to solve real-world problems (Nesher & Hershkovits, 1997). Hence, real-world problems are required to be incorporated into mathematics teaching. Due to rapidly changing conditions around the world, a great number of problems and events affect our personal and professional lives to a large extent. The main skill we should have under these conditions is not to treat these problems superficially (Broudy, 1982). In other words, people who are able to "solve problems" should be trained through a qualified education program. A number of educationalists state that problem solving is highly significant in achieving educational success and it, therefore, should be the main objective of mathematics teaching at all levels of education (Charles & Lester, 1984). Dynamic software is considered as an auxiliary element in this process. One of the dynamic geometry software, GeoGebra is a tool for encouraging and motivating teachers for incorporating geometric constructions (Stupel & Ben-Chaim, 2013).

GeoGebra is free, open source-coded, with multiple representations (e.g., different windows for algebraic and geometrical input) and it offers individualized language options, interactive commentary and Internet options for sharing sources (Hohenwarter, Hohenwarter, Kreis, & Lavicza, 2008). GeoGebra transfers mathematical symbols, graphics, and values via various windows (Aktümen, Horzum, Yıldız, & Ceylan, 2010). Through algebra and graphics windows, it enables views of algebraic and graphical changes on the chart at the same time (Hohenwarter & Jones, 2007). Moreover, it contributes to the modeling of real life problems (Baltaci, 2014; Price & Stacey, 2005). It is a versatile tool for visualizing and objectifying mathematical concepts (Hohenwarter & Jones, 2007; Hohenwarter, Preiner, & Yi, 2007). Also, GeoGebra software expands the dynamic geometry concepts, and adds a new dimension to mathematics and algebra (Dikovich, 2009). Furthermore, some innovations have been introduced with the new versions of GeoGebra (i.e., GeoGebra 3D window). Within this context, it is getting easier to visualize 3D subjects owing to this window (Yıldız, Baltacı & Aktümen, 2012). Having all these functions, GeoGebra is a useful tool in teaching and learning geometry.

Studies on how gifted students solve problems are usually based upon how secondary-school gifted students solve non-routine mathematical problems (Duzakın, 2004; Garofalo, 1993; Sriraman, 2003). Besides, learning geometry involves three cognitive processes: visualization, geometric constructions and reasoning. While various studies are available regarding visualization and reasoning (Baki, Kosa & Guven, 2011; Gutierrez, 1992; Guven & Kosa, 2008; Wong & Bukalov, 2013; Yolcu & Kurtulus, 2010), only a few studies focus on geometric construction. Considering the studies conducted on geometric constructions in the literature, a few studies such as teachers' usage of geometric construction process in lessons (Erduran & Yesildere, 2010), the relationship between the construction activities and students' Van Hiele geometric thinking level (Cheung, 2011; Guven, 2006) and the usage of different construction tools (compass, ruler, dynamic geometry software etc.) in geometric construction activities were examined (Kondratieva, 2013; Kuzle, 2013; Pandiscio, 2002). In Turkey geometric construction activities are available in mathematics and geometry teaching programs starting from primary school to high school; however, several problems arise in the process of these activities (Erduran & Yesildere, 2010). Geometric construction levels of gifted students are not taken into consideration in solving real-world problems in Turkey. Therefore, the current study aims to examine the geometric construction abilities of ninth-grade (15 years old) gifted students in solving real-world geometry problems. In parallel with this, the research questions are;

1. How are geometric construction abilities of ninth-grade gifted students in solving real-world geometry problems with pen and paper?

2. How are geometric construction abilities of ninth-grade gifted students in solving real-world geometry problems with GeoGebra dynamic mathematics software?

METHODOLOGY

In this section, the research method, participants of the study, application process, data collection and analysis are presented.

Research Method

In the present study, a case study method was used to investigate a certain group deeply and to assess the data without any concern about generalization. According to Yin (2003) a case study design should be considered when: (a) the focus of the study is to answer "how" and "why" questions; (b) it is impossible to manipulate the behavior of those involved in the study.

Participants

Gifted students in Turkey receive education in the Science and Arts Centers which are different educational institutions independent from formal school programs. The selection of gifted students for these education centers is composed of six stages, namely: diagnosis, designation for candidacy, preliminary evaluation, group scanning, individual scrutiny and enrolment-placement. Students who succeed in all stages have the right to receive education in those Science and Arts Centers, which means that Turkey is hypercritical concerning selection of gifted students.

The participants were determined through purposeful sampling approach in the present study. Purposeful sampling method is used to identify and select information rich cases (Patton, 2005). While deciding on the participants in the research, the researcher took into consideration the skills of self-expression and ability to volunteer for clinical interview. The study involved six ninth grade gifted students, three males and three female. Five of them have been in the Science and Art Centers since the fourth grade (10 years old), and the last one has been in the Science and Art Centers since the fifth grade (11 years old). In addition, all of the participants were successful students and their grades were all 5 out of 5. All the students also have the skills required to use GeoGebra dynamic software.

Application Process

Before the application process, the gifted students were taught how to use GeoGebra software in 9 days, during holidays, in the Science and Arts Center. During this process, the GeoGebra interface was presented to students and they were informed about how to use GeoGebra software; then they performed a variety of activities. A pilot study was done with two gifted students with the aim of determining the expected problems. Following the pilot study, the problems prepared by the researcher and the questions asked during the clinical interview were checked by two field experts for application after the general revision. The problems are presented in the Appendix.

Students were informed about the purpose of the study superficially before conducting the application. During the application, the problem solving process of each student was examined in detail with a clinical interview. Each of the interviews occupied approximately one hour; all of them were performed in the counseling room, which was quiet enough to relax the participants.

Data Collection

The data included voice records, students' solutions and models on the GeoGebra screen. Before conducting the research, the questions asked in the clinical interview were told to the students. The problems were prepared in accordance with the mathematics curriculum and students' textbooks; besides, three problems were chosen out of these problems by asking the views of mathematics teachers working at the Science and Art Center and two field experts.

Data Analysis

Three problems were asked to the gifted students. During this activity, students were interviewed and their answers were recorded. Following that, the data were presented to the participants in order to avoid misunderstandings.

Before the data analysis, the interviews and the solutions of problems were transcribed verbatim and checked. The related information was given to a field expert. Then, the themes were separately developed by the researcher and a field expert. All the analyses were finalized by bringing together and discussing them. At the end of discussions, the agreed themes were propounded so as to constitute an answer to the research problem. This is a significant requirement referenced in ensuring the validity of the research. Furthermore, Wolcott (1990) stated that explaining the results with direct quotations of the real individuals is important for reliability of the study. Thus, it is tried to create the reliability by giving some of the data directly from the current study.

FINDINGS

The findings were composed of direct quotations, students' solutions, and models on the GeoGebra screen.

First Problem Solving Process

Four of the gifted students answered the problem correctly in writing. As an example, G1's expressions are as follows:

R: How do you think to solve the problem?

G1: We found the height is 80 cm. The site of the angle is four divided by five.

R: Yes.

G1: The cover's height is 60 cm. Because this length is 10 cm and that one is 10 cm too (being parallel to edges and 10 cm apart from these edges). I found 60 if 80 minus 20.

R: You said the height of the cover is 60 cm.

G1: Yes. The edges are in parallel to each other. Therefore, the angles and the sine are equal.

R: How?

G1: They are corresponding… 60 divided x equals 80 divided by 100. So x is 75 cm.

(G1: Gifted student 1, R: Researcher)

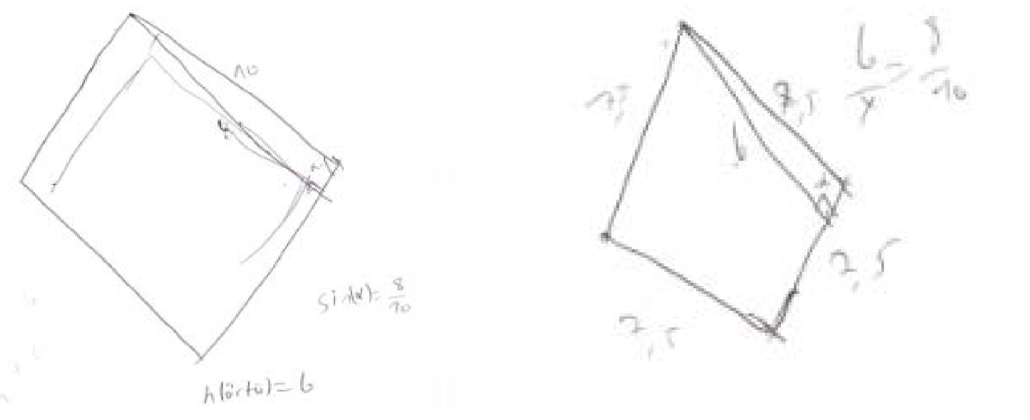

FIG. 1. G1's solution in paper and pencil environment.

All students were successful in modeling with an appropriate rate. And, some of the gifted students' ways of thinking, while modeling rhombus via using special triangle, were different. G4's expressions are presented as an example below:

R: How can you model this problem on the software?

G4: I'll model it with an appropriate rate as it is real world size.

R: How?

G4: Instead of 100 cm, it can be 10 cm. It will be 8 cm for the height, then.

R: Let's do it.

G4: I won't model the rhombus in these rates. I can do something else.

R: So how?

G4: The height is 8 and one of the edge is 10. Then we can use 6-8-10 special triangle.

R: Do it please.

G4: I modeled it by using parallelism. This is our cover.

Different from the others, G2 and G3 constructed the problem as following and accomplished.

G2: The starting point of the line segment is (6,4). In order to be 10 unit away, lets mark (16,4).

R: Yes.

G2: The height is 8, so I marked (16, 12). We can find the table by compounding these points and line segments…

G6 made several mistakes during the modeling, but it was observed that they corrected the mistakes immediately subject to the feedback. G6's model is presented in Figure 2.

G6: There is a length button between the two points. If I am not mistaken, I will find out one of the edge's length of the cover.

R: OK. Let's find it.

G6: 12.51.

R: So, you say one of the edge's length of the cover is 12.51.

G6: Yes. I want to find out whether the other lengths are the same or not.

R: Let's see.

G6: I think it is enough to find out one edge more. But the length is 12.4 in here.

R: Where is the error?

G6: I made a mistake while scrolling through. I haven't taken it as parallel.

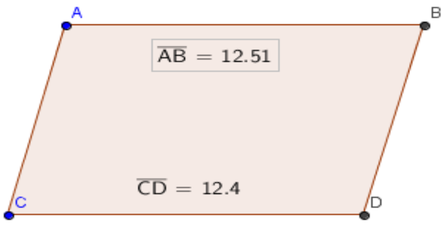

FIG. 2. G6's incorrect model.

Then, the students modeled the line which is short of 10 cm and parallel to the edges with the intersection of perpendicular lines and circles correctly. This process was exemplified by G5's expressions as follows:

G5: Parallel to this, it is 1 unit away. How can I measure it? Maybe from the circle.

R: How?

G5: We cannot draw perpendicular line to line segment in the program. For this reason, I will use the circle. I am drawing 1 unit radius circle on one of the points of rhombus. After that, I am drawing a parallel line through the center of the circle like that. After doing the same thing for the other edges, I will form the cover by taking the intersections of the lines.

The situation witnessed above gives clues regarding gifted students' geometric construction abilities since modeling of intended situation available in the problem is determined only when some mathematical concepts and the software are used coordinately. Then, all the students solved the problem correctly. To illustrate, G1's model for the first problem is presented in Figure 3.

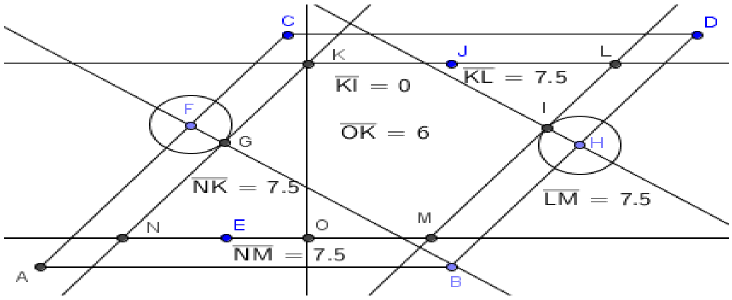

FIG. 3. G1's model through solution.

Next, all students checked the correctness of the final result by means of graph and algebra windows of the software.

Second Problem Solving Process

It was determined that gifted students visualized the models when the problem was understood. It indicates the ability of gifted students while doing the mathematical processes in their minds correctly and practically. G3's expressions are presented as an example:

R: Can you make a prediction about the result of the problem?

G3: The height is 30 cm. Radius is 20. If we take 3 for the π, the answer is 1200x30=36000 cm$^{2.}$

R: Why do you think so?

G3: Did I make a mistake? I think not.

As pointed out above, all of the students solved the second problem correctly in their minds. Still, how the geometric construction occurred through GeoGebra software is a question of debate expected to be answered. Hence, students were asked to model via using the software. It was observed that all the students reduced the figure at the appropriate rate. For instance, the models of G2 (Figure 4), G4's (Figure 5), and G5's expressions during the process of geometric construction are presented below:

G5: I take 4 unit cube to modeling in the software.

R: OK.

G5: I will draw a circle for the cylinder.

R: Where?

G5: On the base of the cube.

R: How can you draw?

G5: Firstly, I form a line segment like that. The middle point would be like that via the software.

R: Then, what are you going to do?

G5: (thinking) To form the radius, I will draw perpendicular from the center to the edge of the cube. This would be like that via perpendicular line toolbar of the software. Then, for the radius, I state the line segment and hide the line.

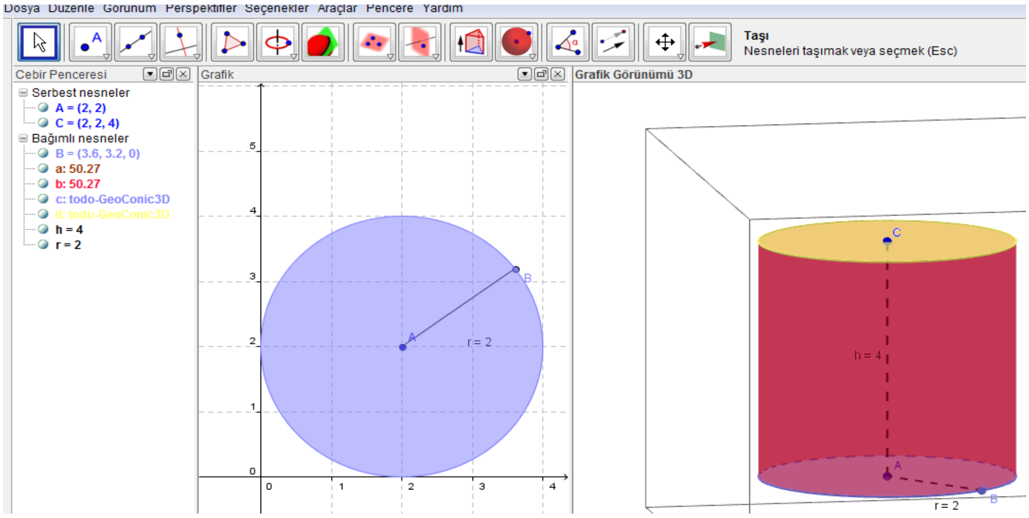

FIG. 4. G2's model.

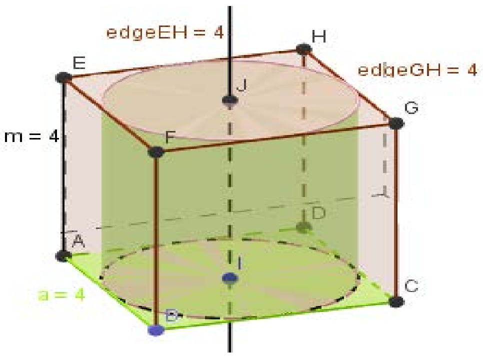

FIG. 5. G4's model.

As can be observed in the figures above, it was not easy to construct the biggest cylinder into the cube. During the construction process, the gifted students had to use their prior mathematical knowledge so as to understand the toolbar presented by the GeoGebra software, and so they accomplished. Furthermore, students were encouraged to use the 3D window easily even though GeoGebra dynamic software had been already presented to them, which ensures that gifted students have high capacity for learning. Moreover, it was confirmed that the students made observations through rotating and coloring the models for a long time. It clarifies that practical and functional use of 3D window of the GeoGebra software had an effective role in this situation.

Third Problem Solving Process

It was identified that all of the students could not make reasonable predictions related to the result of the third problem. The expressions of G3 and G6 for this situation are given below:

R: Can you make prediction about the result of this problem?

G3: Let me find out the average result of the problem.

R: How?

G3: At first, I will find the quarter of it. So I divide it 4.

R: What is divided by 4?

G3: The big one, it is 75, and if this part is 50, then it becomes 125.

R: Can you make prediction about the result?

G6: What is π?

R: 3.

G6: It is about 190.

R: Why?

G6: The addition of areas of the half circle whose radius is 10 and the quarter circle whose radius is 7, is about 190.

It was observed that the students did the operations successfully and fast in their minds while making predictions as above; nevertheless, the problem is that the students failed in terms of the inappropriate planning related to the solution of the problem. In any case, G2, G5 and G6 modeled and solved the third problem incorrectly in writing. However, G2, G5 and G6 modeled this problem correctly by specifying the previous mistakes in the software. Accordingly, it refers to the fact that these students interpreted and evaluated the mathematical operations and the results from the graphic and algebra screens of GeoGebra successfully. The model and evaluation of G2's expression (Figure 6) are presented below:

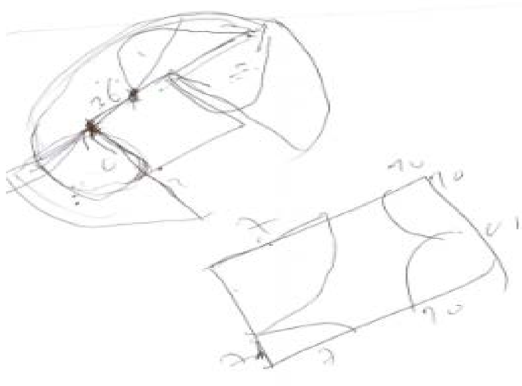

FIG. 6. G2's incorrect model on paper.

G2: Ohh. I realize it. I made a mistake. It does not surround this part.

R: Why?

G2: Because the rope comes here but not there. I made a mistake here.

R: What kind of mistake?

G2: Because the sheep goes around 7 m. It can come here at most, and then it can go 3 m away, so it cannot come here.

Then, they completed the geometric constructions using the software as in G5's case:

G5: Now, I will form the rectangle. I am drawing 6 unit line segment with the software toolbar. But this toolbar draws the line segment on the right side. How can I draw rectangle?

R: Think a bit.

G5: Maybe, firstly, I will draw a vertical line at the end of the line segment to the same line segment. Then, to state the 4 unit distance, I will use the center and radius length and circle menu. I will take the intersection of the line and circle's end point. Then, I will hide the line which is in the same direction with this line. The rest is easy.

All the gifted students created the rectangles as in G5's construction process above. During this process, students used the features of line segment, vertical line, circle and parallel line. In this way, students succeeded in constructing the rectangle through using mathematical information coordinately. Then, all students found the result by drawing circles where the sheep could walk around.

DISCUSSION

It was determined that the gifted students were able to solve all the problems by means of connecting the rich features of the GeoGebra software with the personal abilities with an elaborative analyzing process. In parallel with this finding, Baki, Yıldız and Baltacı (2012) demonstrated that the gifted students solve real-world problems easily with different ways of thinking provided that they are given efficient opportunities. Additionally, Rogers (2002) stated that there should be special classrooms for gifted students, and in this way, there could be a competitive environment as well as opportunity for sharing knowledge with their peers. As indicated by Anabousy, Daher, Baya'a and Abu-Naja (2014), Gonzàlez and Herbst (2009), and Santos-Trigo and Cristóbal-Escalante (2008), the gifted students discover by experiencing through the GeoGebra software. Contrary to some other software, diversity of GeoGebra software plays a significant role in the study as GeoGebra is free and integrates algebra and analysis in a single package (Tatar, Akkaya, & Kagızmanlı, 2014). Hence, in the current study, all the gifted students constantly questioned and checked the models. Consequently, as discussed in Ellerton's (1986) study, the gifted students are quicker at realizing their errors during problem solving.

All the gifted students were successful in modeling the figures by reducing them with an appropriate rate in the software. Besides, as Wieczerkowski, Cropley, and Prado (2000) stated, solving the problems expeditiously or memorizing the symbols, numbers, and formulas are not accepted as indications of being gifted. Similarly, Wang (1989) and Ellerton (1986) concluded that gifted students were good at effective planning. In conclusion, the gifted students participating in this study succeeded in their thinking processes while trying to solve real-world problems. Shore and Dover (1987) expressed that the thinking process of the gifted students is different, because they need more thinking abilities in order to evaluate what they learn (Amick, 1985). Thus, the gifted students think analytical, inductively and deductively in solving hard and complicated problems (Holton & Gaffney, 1994).

It was found that students made observations for a long time after they set models via GeoGebra software. It is likely that the practical and functional use of the 3D screen of the GeoGebra software had an effective role in this situation. Under the strength of this finding, it is claimed that the gifted students were willing to work systematically. This situation is not in parallel with the study of Heinze (2005) who found that these children stood out in their high ability to verbalize and to explain their solutions. However, Gorodetsky and Klavir (2003) stated that the gifted students demonstrated reflection in action during problem solving.

Students could not decide where to commence a construction activity at first (Erduran & Yesildere, 2010). However, all of the students managed to overcome this problem by effectively using their mathematical abilities in the construction process. While solving the first problem, some of the gifted students' way of thinking in modeling rhombus by means of using special triangle was different from the others. Likewise, Sisk (1987) and Scruggs and Mastropieri (1985) stated that the gifted students can find different solutions and make creative interpretations. Besides, the gifted students were able to use the information, learned in the geometric construction process, when required, successfully since the modeling of intended situations in the problems occurred only through using some mathematical concepts and the software coordinately. This finding is compatible with the findings of Laborde, Kynigos, Hollebrands and Strasser (2006); in that learning is not quite easy in the environments where dynamic geometry construction is used as the students reconstituted their own geometry knowledge. In addition, Baltaci and Yildiz (2015) and Sendova and Grekovska (2005) pointed out that building a computer model of a construct is motivated to elaborate one's knowledge in mathematics.

All in all, it was verified that gifted students think more creatively during geometric constructions thanks to the functionality and practicability of the GeoGebra software. As Mainali and Key (2012) point out, students start to explore concepts as a result of using the dynamic software. Thus, dynamic software is expected to have a significant place in the education of gifted students. It is essential for countries to improve the abilities of gifted students. As discussed previously, in Turkey, the construction activities are carried out solely via compass, ruler and quadrant, in order to draw models and let them see what they did during the construction process, in primary and secondary school education. Nevertheless, geometric construction works are of value not only for developing the skills of using drawing tools, but also for developing the students' ability of thinking deeply with regard to their construction processes (Cherowitzo, 2006). Cheung (2011) found that the experience of geometric construction inspires students more for thinking deeply. Thus, the teachers, trying to give fruitful lessons, should use GeoGebra dynamic mathematical software actively in order to develop their students' thinking abilities.

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Appendix-1. The Problems Used in This Study

First Problem: Tuğba's mother wants to sew a cover for the rhombus kitchen table. One of the edges is 100 cm and the area is 80 dm^2. The cover needs to be parallel to the edges of the table and 10 cm remote from each edge. But she cannot find the appropriate size of this cover. Can you help her to find the size of the cover?

Second Problem: Mehmet finds a cube block whose one edge is 40 cm. He wants to cut this cube from the point which is remote 10 cm from the upper base as parallel to the lower base. He also, wants to form a cylinder from the bigger piece of two. What is the volume of the cylinder block?

Third Problem: While Akif was going to his village, he met his uncle. He saw that his uncle bound a sheep to the cottage, and he was curious about how far the sheep could graze in this area. To find an answer for this question, he decided that the size of the cottage was 4 and 6 meters, the sheep could not jump from the cottage and the sheep was bound with a 10 m rope from the midpoint of the longest edge of cottage. How does Akif find the grazing area of the sheep?